# Ray Theory Applied to Stability, Fluctuations, and Time-reversal in Deep Water Acoustics

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### LONG-TERM GOALS

The long-term goals are: 1) to take advantage of the ever-changing ocean environment's effects in order to provide a more complete understanding of long-range acoustic pulse propagation, 2) to understand the extent of fundamental limitations on ray-based acoustic tomography; of particular interest is the breakdown range of semiclassical methods, 3) to develop the theory of the statistical fluctuations of the wavefield, 4) to develop a full ray theory of the many aspects of time reversed wave propagation, and 5) to address important basic physics issues that arise in the ocean problem, but within a more general wave propagation in random media context.

### **OBJECTIVES**

There are four primary scientific objectives of this work: 1) to begin developing a geometric acoustics theory that addresses parametrically varying ocean environments in the presence of ray chaos, determines what information survives under such conditions, and determines how to extract it, 2) to develop the geometric acoustics theory of wavefield fluctuations, 3) to determine the sensitivity of acoustic wavefields to relevant ocean environment parameters thereby connecting the scale of changes in the ocean to range scales of wavefield correlation decay, and 4) to develop a geometric acoustics theory of time reversed acoustic wave propagation.

### APPROACH

We consider acoustic propagation problems that allow for parabolic equation description. Advantage is taken of new semiclassical approaches to approximate time-evolving wavefields in systems possessing classically chaotic analogs. The methods rely on wave packets, canonical transformations to action-angle variables, canonical perturbation theory, heteroclinic

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Form Approved OMB No. 0704-0188 orbit summations, and have been shown to be remarkably accurate in spite of relying on highly unstable chaotic trajectories. The approach is similar in spirit to the van Vleck approximate propagator, and the Gutzwiller trace formula. From this starting point, we consider systems whose governing equations can be expressed as varying with respect to a parameter; this can model, for example, a time-changing internal wave configuration. To study response and sensitivity, it is fruitful to apply perturbation theory to describe the changes arising in ensembles of classical trajectories underlying the wavefields. We compare semiclassical predictions with 'exact' numerical wavefield calculations.

### WORK COMPLETED

The work completed this year is part of two collaborations. The first work is with a WSU research assistant professor, Nicholas Cerruti, and Ph.D. student Katherine Hegewisch. Before investigating the fundamental limitations on ray methods in the presence of chaos, it is necessary to address another issue. An acoustic signal, which has some minimum wavelength component, would intuitively be expected to refract much less from fine structures in the ocean's internal wave field once these structures were shorter than this wavelength. On the other hand, geometrical ray tracing methods are sensitive to infinitely fine structures in the internal wave modeling. This difference in sensitivity harms the ray/wave correspondence, and it is a separate issue from that of chaos. Hence, a proper model seeking agreement between ray methods and wave propagation must, at a minimum, filter out the fine oscillations of the internal wave field. Starting from an efficient numerical scheme for generating the internal waves introduced by Brown and Colosi, which reproduces the Garrett-Munk spectrum, we introduce a smoothing technique that removes the unphysical portion of the internal wave modeling. The key is to find a smoothing that does not significantly alter the propagated wave field, yet eliminates as much of the "micro-folding" phenomena that was discussed in Simmen, Flatte and Wang. We have propagated wave fields to various ranges and with various smoothings and measured the extent to which the fine structures can be neglected af a function of source frequency.

The second work is with Jian Huang and Eric J. Heller, both of Harvard University. For the purpose of understanding time-reversed wave fields better, we investigated the effects of local perturbation on wave propagation for an open system comprised of a large number of randomly placed identical scatterers. We first derived formal expressions for the changes that take place when a single scatterer is removed, moved, etc., whose direct application is computationally feasible only for ballistic transport at weak scattering limit. In the diffusive regime of strong scattering, we looked at the diffusive transport of perturbative scattering in the recent acoustic experiments on time-reversal focusing of Tourin, Derode, and Fink.

# RESULTS

In the first project, we have propagated wave fields up to 4,000 km through various realizations of the internal wave field. Let  $J_{max}$  denote the highest contribution in the summation over vertical modes. We define a correlation measure as the squared overlap between the acoustic waves propagated through the internal wave field created with cutoff at  $J_{max}$  and the acoustic waves propagated through the internal wave field including the first 200 modes. The initial conditions are identical for all calculations. See Fig. (1). A

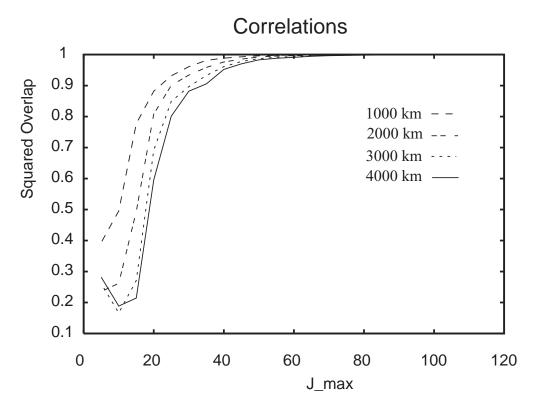


FIG. 1: Correlations as a function of  $J_{max}$  for propagation up to 4,000 km. The source frequency is 75 Hz. The proximity to unity once  $J_{max}$  exceeds roughly 50-60 shows that the higher modes are not significantly influencing the propagating acoustic waves for this frequency. The optimal cutoff increases naturally with increasing source frequency.

value close to unity indicates that the truncation of the internal wave field calculation had little to no affect on the propagated acoustic wave.

We further investigated the affects that smoothing of the fine scale structures in the internal wave field had on the propagation as a function of the sound source frequency. In Fig. (2), we show the affect our smoothing method has on a realization of the internal wave field. We find that the acoustic waves are not sensitive to structures considerably longer than their wavelength due to the horizontal dominance of the propagation. For example, a 75 Hz source has a 20 m wavelength, but the propagated sound was not sensitive to a scale

# **Smoothing Effects**

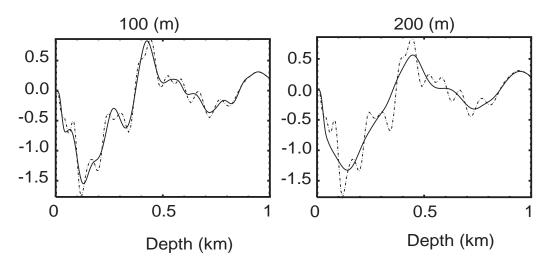


FIG. 2: Illustration of the effect of smoothing on a realization of the ocean's internal wave field. The vertical axis represents the sound speed change,  $\delta c/c_0$  in units of  $10^{-3}$ . The dashed line is one realization of the internal wave field using all modes up to  $J_{max} = 200$ . The solid line is the result after smoothing. The effective cutoff scale is indicated above each panel.

smaller than roughly 100 m.

In the second project, we reproduced several experimental results. Numerical simulations of time-reversed focusing in a 2D random s-wave scattering system also confirm both the enhancement of refocusing and the generality in the deterioration pattern of a square root dependence on the number of scatterers removed before reversal.

# IMPACT/APPLICATION

The work is aimed at understanding the predictability and/or other limitations of ray methods in the presence of unstable dynamics. In addition, parametric variation, once understood, is often found to be one of the only successful ways of deducing otherwise difficult-to-ascertain information about complex systems such as the ocean environment. It may lead to new ocean acoustic tomography techniques.

# **TRANSITIONS**

It is too early to discuss how these results will eventually be used by others.

### RELATED PROJECTS

Additional work currently underway, but not described in this report, involves collaborations with the following individuals: M. Brown (RSMAS-AMP).

## REFERENCES

- M. A. Wolfson and F. D. Tappert, "Study of horizontal multipaths and ray chaos due to ocean mesoscale structure", J. Acoust. Soc. Am. **107**, 154–162 (2000).
- P. F. Worcester, B. D. Cornuelle, M. A. Dzieciuch, W. A. Munk, B. M. Howe, J. A. Mercer, R. C. Spindel, J. A. Colosi, K. Metzger, T. G. Birdsall, A. B. Baggeroer, "A test of basin-scale acoustic thermometry using a large-aperature vertical array at 3250-km range in the eastern North Pacific Ocean", J. Acoust. Soc. Am. **105**, 3185–3201(1999).
- J. A. Colosi and M. G. Brown, "Efficient numerical simulation of stochastic internal-wave-induced sound-speed perturbation fields", J. Acoust. Soc. Am. **103**, 2232–2235 (1998).
- J. Simmen, S. M. Flatté, and G.-Y. Wang, "Wavefront folding, chaos, and diffraction for sound propagation through ocean internal waves", J. Acoust. Soc. Am. **102**, 239–255 (1997).
- A. Tourin, A. Derode, and M. Fink, "Sensitivity to Perturbations of a Time-Reversed Acoustic Wave in a Multiple Scattering Medium," Phys. Rev. Lett. 87, 274301 (2001).

# **PUBLICATIONS**

- N. R. Cerruti and S. Tomsovic, "A Uniform Approximation for the Fidelity in Chaotic Systems", J. Phys. A: Math. Gen. **36**, 3451 (2003); nlin.CD/0210030.
- M. G. Brown, J. A. Colosi, S. Tomsovic, A. L. Virovlyansky, M. A. Wolfson, and G. M. Zaslavsky, "Ray dynamics in long-range deep ocean sound propagation," J. Acoust. Soc. Am. **113**, 2533 (2003); nlin.CD/0109027.
- F. J. Beron-Vera, M. G. Brown, J. A. Colosi, S. Tomsovic, A. L. Virovlyansky, M. A. Wolfson, and G. M. Zaslavsky, "Ray dynamics in a long-range acoustic propagation experiment," J. Acoust. Soc. Am. **114**, 1226 (2003); nlin.CD/0301026.